A STUDY OF THE DOWNWARD FLOW OF A LIQUID FILM ALONG A VERTICAL SURFACE WITH ATTENDANT HEAT TRANSFER

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This report deals with a theoretical and experimental study concerning the measurement of local values of mean film thickness throughout the film length and with theoretical relations derived for determining the film thickness in the laminar-wave as well as in the turbulent region. The experimental studies have also covered local characteristics of wave motion on the film surface as well as the velocity field in the continuous layer, the local heat-transfer coefficients along the height of a vertical channel, and a critical relation for determining them has been derived.

Nusselt [1, 2] was the first one to thoroughly study the flow of liquid films along vertical surfaces under the influence of gravity. He has derived theoretical relations for the film thickness, the velocity distribution across it, and the heat-transfer coefficients in a laminar steady flow, where the forces of gravity are in equilibrium with the forces of friction. Later studies [3, 4, 5] have shown that under real conditions and already at low rates of liquid flow there appear spatial waves in an irregular pattern on the film surface. A theoretical study of laminar liquid flow has been conducted in [3, 4] with these waves assumed to be planar and sinusoidal.

The region of complete turbulence was studied experimentally [5, 6, 7, 8, 9] in relatively short tubes. The object of these studies was to determine the mean-over-the-height equilibrium thickness of a film and to find empirical relations for calculating it. Furthermore, changes in the wave character on the surface of a film flowing along a vertical wall were referred to qualitatively without any quantitative generalizations. Changes in the flow conditions along the channel height should certainly be reflected in the local values of the heat-transfer coefficient. However, all known methods based on empirical relations yield only the mean-over-channel-height coefficient of heat transfer to a descending liquid [7, 8, et al.] and are not applicable to conditions different than those in a particular experiment. Even in [7], where local values of the heat-transfer coefficient had been determined directly and considerable variations along the height had been found under several sets of conditions, the evaluation was based on mean values. In the conclusion, several critical equations have been derived, each one valid for a narrow value range of basic parameters.

The report covers a theoretical and experimental study of local film flow characteristics in the laminar-wave region ($Re_\delta < 400$) as well as in the complete turbulence region ($Re_\delta > 400$) and an experimental study of local heat-transfer coefficients.

When changes in the mean-over-channel-height film thickness are determined theoretically, it is assumed that the liquid enters through the orifice of a nozzle at a velocity uniformly distributed across the orifice. As the liquid flows, there builds up a boundary layer. Outside this layer the liquid is accelerated by the force of gravity and is driven away from the wall. This continues until the boundary layer occupies the entire film thickness over a distance $x = L_{bound}$ from the inlet. Where $x > L_{bound}$, the liquid either accelerates or decelerates until the forces of gravity and the forces of friction are in equilibrium. Assuming that the profile of relative velocity (local velocities referred to the velocity at the film boundary) at $x > L_{bound}$ remains similar and can be described by a third-order polynomial where $Re_\delta < 400$ but by a

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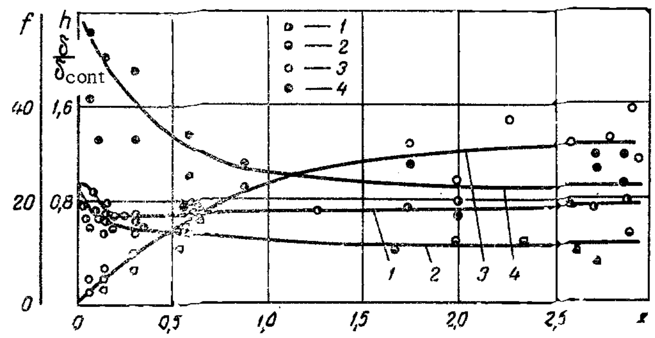

Fig. 1

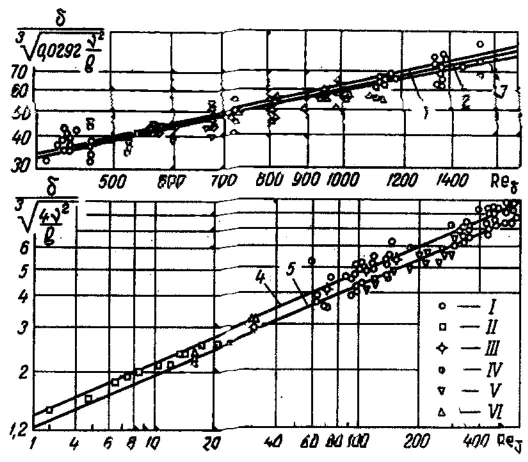

Fig. 2

Fig. 1. Variation of local-along-channel-height characteristics: 1) δ; 2) δ_{cont}; 3) h; and 4) f, sec^{-1}.

Fig. 2. Dimensionless film thickness as a function of the Re_δ number: 1) according to the data in [8]; 2) according to Eq. (3); 3) according to the data in [6]; 4) according to Eq. (2); 5) according to the Nusselt equation [5]; I) test data MVTU; II) data [3, 4]; III) data [11]; IV) data [10]; V) data [12]; VI) data [13].

power-law relation in the region of complete turbulence, we obtain the following expressions for the local-along-channel-height values of mean film thickness:

$$\frac{1}{6B\delta_{\lim}} \ln A \frac{\delta_{\lim}^3 - \delta^3}{(\delta_{\lim} - \delta)^3} + \frac{1}{b\delta_{\lim}^2 \sqrt{3}} \operatorname{arctg} \frac{2\delta + \delta_{\lim}}{\delta_{\lim}\sqrt{3}} - B = x, \tag{1}$$

where

$$A = \frac{(\delta_{\lim} - \delta_0)^3}{\delta_{\lim}^3 - \delta_0^3}; \quad B = \frac{1}{b\delta_{\lim}^2 \sqrt{3}} \operatorname{arctg} \frac{2\delta_0 + \delta_{\lim}}{\delta_{\lim}\sqrt{3}}$$

For laminar flow ($Re_\delta < 400$)

$$\delta_{\lim} = \sqrt[3]{\frac{4\nu^2}{g} Re_\delta}; \quad b = \frac{7}{8} \cdot \frac{g}{Re_\delta^2 \nu^2}. \tag{2}$$

For turbulent flow ($Re_\delta > 400$) with the exponent in the power law representing the velocity in dimensionless coordinates n = 1/7, we have

$$b = \frac{63}{64} \cdot \frac{g}{Re_\delta^2 \nu^2}, \quad \delta_{\lim} = \sqrt[3]{0.0292 \frac{\nu^2}{g} Re_\delta^{\frac{7}{4}}}. \tag{3}$$

In Fig. 1 we compare the variations of the mean calculated and the measured thickness for a water film along a vertical tube with a trickle intensity Γ = 8.22 N/m · sec at a film temperature t_f = 15°C. A close agreement between calculated and test results is evident here. The initial range within which δ differs from δ_{lim} considerably is small and depends on the liquid flow rate. It does not exceed 100-300 mm for all the trickle intensities considered here.

The experiments were conducted on a specially equipped test stand, whose active part consisted of a vertical tube 3 m long with a 61.2 mm outside diameter and made of 1Kh18N9T steel. From an overhead tank with a free surface the working liquid (distilled water) was flowing down due to gravity along the outer surface of the experimental section. The orifice in these tests was 1 mm wide. The tubes were heated by hot water flowing inside them in the upward direction. The temperature of the liquid during isothermal film flow tests was varied within the 4-36°C, which corresponds to the most drastic variation in its viscosity. The weight intensity of the trickle in these tests was Γ = 0.881-17.35 N/m · sec and Re_δ varied between 65 and 1560.

The local heat-transfer coefficients were studied at a liquid inlet temperature of 6-65°C. The intensity of trickle in these tests was varied from 4.82 to 12.74 N/m · sec.

The character of film flow and its local parameters were studied by the capacitance method. The transducer was a capacitor whose one plate consisted of the active tube portion and the second plate consisted of a movable metallic plate 2.5×4 mm^2 large mounted in such a way that the separation between both plates remained always much greater than the maximum wave height in the test region. The capacitance of such a capacitor varies as a function of the instantaneous liquid film thickness between its plates. The capacitance variations were measured with an E8-1 instrument, which has a 10^{-4}-50 pF ± 1-5% range, in series with a null indicator connected to an N 700 loop oscillograph. From the oscillograms obtained in this way it was possible to determine δ, δ_{cont}, δ_{wave}, and the frequency of wave motion f. The mean film thickness was determined by averaging the instantaneous thicknesses over a period of 1 sec. The frequency was counted directly on the oscillogram, disregarding the small-scale pulsations at a 150-200 sec^{-1} frequency.

The velocity field in a continuous layer was studied by means of a thermoanemometer with a tungsten filament 10 μ in diameter and 2-3 mm long operating on alternating current on the constant-resistance principle. The thermoanemometer filament was mounted in a horizontal position and was shifted by means of a micrometer screw relatively to the wetted surface and perpendicularly to it. The velocity averaging effect due to the straight filament approaching a curved surface was overlooked here, which is entirely permissible under such particular test conditions (diameter of the active tube section approximately 61 mm, length of filament 2-3 mm).

The heat-transfer coefficient was calculated from the expression

$$\alpha = \frac{q}{\Delta t}\,. \qquad (4)$$

The local thermal flux density q was measured over a 200 mm long section. In order to determine the quantities which appear in Eq. (4), the temperatures of the heat carrier t_h, of the wall t_w, and of the film t_f were measured with Chromel–Copel thermocouples which had been installed at respective locations along the channel height. The thermocouple readings were taken with PP 63 Class 0.05 instruments. Temperature t_h was measured with bare-junction thermocouples placed in the stream through a sleeve 2 mm in diameter. The wall temperature was measured with thermocouples which had been formed by two bare electrodes 0.2 mm in diameter and separately welded to the wall in a normal position with a 2-3 mm distance between them. A systematic error was incurred here in the temperature determination on account of heat leakage along the thermoelectrodes. The magnitude of this error was calculated on a computer and an appropriate correction was added to the measured value of wall temperature. In this way, the impossibility of locating the hot junction was overcome and maximum simplicity in the thermocouple preparation was ensured. Perturbations caused by the presence of electrodes in the stream were assumed to be negligibly small in comparison with the total perturbations due to wave flow. Temperature t_f was measured with a thermocouple having a bare junction approximately 0.1 mm away from the wall. It was taken into consideration here that, as a result of much displacement occurring during wave motion, the temperature remains almost constant across the film thickness – except for a very thin layer at the wall [7, 8]. The diameter of the thermocouple bead was 0.2-0.3 mm.

The flow rates of the hot heat carrier and of the liquid in the film were measured with standard and precalibrated flow diaphragms.

The thermal flux was calculated on the basis of the heat supplied to the hot carrier liquid and was checked on the basis of the heat received by the liquid in the film. The imbalance never exceeded 4-5%. All measurements were performed at 12-15 sections along the active region.

The authors' experimental data and the relations (2), (3) derived by them theoretically are compared with the experimental data obtained in [2, 3, 4, 10, 11, 12] as well as with the theoretical relation derived by Nusselt [2] and with the empirical relations in [6, 8], all shown together in Fig. 2 where δ-graphs are plotted for the film flow of a trickling liquid in the laminar-wave and in the turbulent region. Relation (2) agrees sufficiently well with the experimental data in the studies referred to and, structurally, it corresponds to the Nusselt equation, but it differs from the latter by a constant factor and for the same starting conditions, therefore, δ will be approximately 10% higher than according to the Nusselt equation in [2].

As is indicated in Fig. 2, however, the test data in [2, 3, 4, 10, 11] for laminar and for wave flow as well as our results over most of the entire range yield higher values than the theoretical relation in [2]. The test data in [12] are interpreted best in terms of the theoretical Nusselt relation.

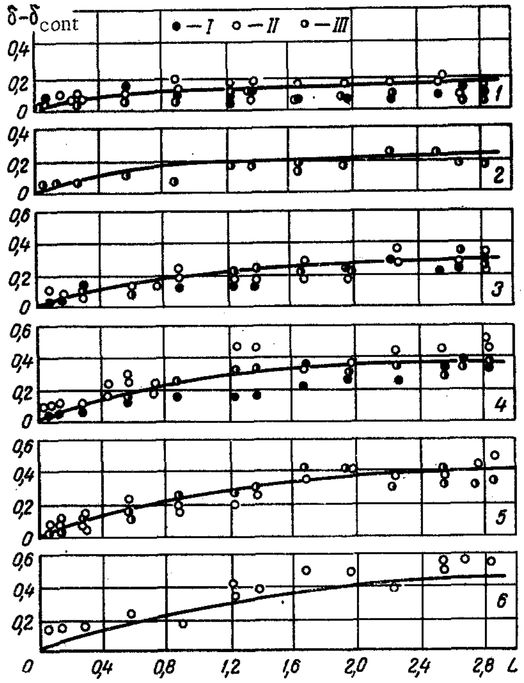

Fig. 3. The quantity $\delta - \delta_{cont}$ as a function of the path length traversed by the film at various trickle intensities: 1) $\Gamma = 0.992$; 2) 3.33; 3) 4.87; 4) 8.22; 5) 9.65; and 6) 11.35 N/m · sec. Solid line corresponds to Eqs. (6), (7), and (8). Experimental data MVTU (Moscow Technical College): I) $t_f = 5$; II) 15; III) 35°C.

Within the turbulent flow region ($Re_\delta > 400$), according to Fig. 2, curve 2 representing the theoretical relation (3) agrees closely with the empirical relations derived in [6, 8] and it also very well describes our experimental data.

Waves of various amplitudes and frequencies appeared on the film surface under all flow conditions which have been studied. A slight ripple was noted already immediately near the entrance into the active region. As the liquid film moved along the vertical surface, the wave pattern changed. This is illustrated in Fig. 1 showing the characteristic variation of δ, δ_{cont}, $h = \delta_{wave} - \delta_{cont}$, and f along the height of the experimental section for $\Gamma = 8.22$ N/m · sec and at a liquid temperature $t_f = 15$°C, which corresponds to complete turbulence ($Re_\delta = 730$). The mean thickness of the continuous layer δ_{cont} changes along the channel and tends toward a limit which depends on the intensity of trickle Γ and on the physical properties of the liquid. The stable region is much larger than for δ and it increases with Γ. The limits of δ_{cont} can, with sufficient accuracy, be found from the approximation

$$(\delta - \delta_{cont})_{lim} = 0.42 \cdot \Gamma^{0.55}. \tag{5}$$

It follows from Fig. 1 that the height of a wave crest increases with the distance from the inlet. This increase becomes greater with a greater trickle intensity. The rate at which h increases becomes gradually smaller and tends toward a definite value which depends on Γ as well as on the liquid temperature. The frequency of waves also depends on the path length traversed by the film and on the trickle intensity. The maximum frequency f is attained immediately at the inlet (see Fig. 1) and it increases with greater Γ. As the film moves down the channel, the frequency drops to a certain value (18-24 sec^{-1}) independent of Γ.

A comparison of $\delta - \delta_{cont}$ values (Fig. 3) shows that this quantity is almost independent of the liquid temperature and is determined by the trickle intensity Γ as well as by the path length x traversed by the film. In $[(\delta - \delta_{cont})/0.8(\delta - \delta_{cont})_{lim}; x/L_{0.8}]$ coordinates this relation can be approximated by a general curve whose equation is

$$\frac{\delta - \delta_{cont}}{0.8(\delta - \delta_{cont})_{lim}} = \log_a \left(\frac{x}{L_{0.8}} + 1\right), \qquad (\delta - \delta_{cont})_{lim} = \delta_{lim} - \delta_{cont}. \tag{6}$$

The following relation was found suitable for determining $L_{0.8}$:

$$L_{0.8} = 1.85\,\Gamma^{0.397}. \tag{7}$$

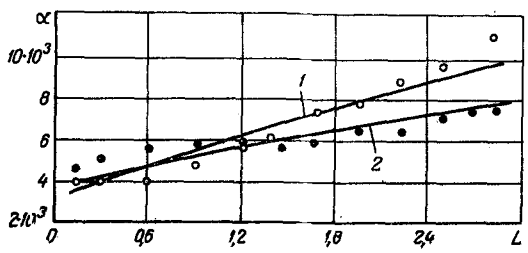

Fig. 4

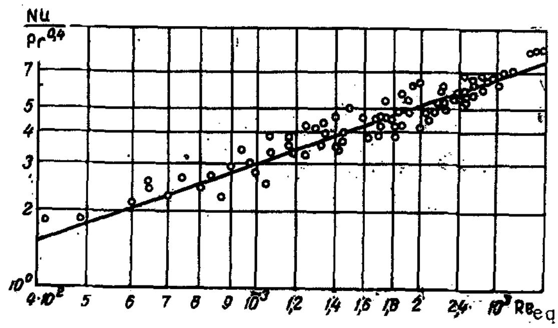

Fig. 5

Fig. 4. Variation of the local heat-transfer coefficient α (W/m^2·deg) along the experimental channel height L (m), for Γ = 8.13 (1) and 12.75 N/m·sec (2). Points indicate test values; solid lines correspond to relation (12).

Fig. 5. Nu/ Pr$^{0.4}$ as a function of Re_{eq}.

The base a of the logarithm in expression (6) is defined by the linear relation:

$$a=0.45\left(\frac{x}{L_{0.8}}+1\right)+1.1. \tag{8}$$

The values of $\delta - \delta_{cont} = f(L)$ determined from expressions (6) and (8) are plotted with solid lines in Fig. 3.

The relations obtained for the local values of δ and δ_{cont} are accurate enough to be used also for isothermal flow, as has been verified experimentally, if Re_δ and ν are determined by the temperature at a given section.

The velocity of the liquid could be measured only in the continuous-layer region. The mean-over-the-section velocity values found by test agree closely with the values calculated from the flow rate and the mean thickness δ_{cont} of the continuous layer at a given section; they are also much higher than the values calculated from the flow rate and the mean thickness δ. Thus, the magnitude of the continuous layer is becoming the criterion for the actual velocities, including the velocities which apparently should be considered in the evaluation of heat-transfer processes.

The variation of the heat-transfer coefficient along the channel height, according to two tests, is shown in Fig. 4. In all the modes which have been studied one observes an increase of α as the film flows down the channel wall. The results of the experiments have been evaluated in terms of the relation:

$$\mathrm{Nu} = A\,\mathrm{Re}_{eq}^{m}\,\mathrm{Pr}^{n}. \tag{9}$$

The mean film thickness δ was taken as the characteristic dimension. The Nusselt number here was $Nu = \alpha\delta/\lambda$.

The film temperature was taken as the characteristic temperature. The mean velocity in the continuous layer v_{cont} was taken as the characteristic velocity. This would make the equivalent Reynolds number

$$\mathrm{Re}_{eq}=\frac{u_{cont}\delta}{\nu}. \tag{10}$$

It is not difficult to ascertain that $Re_\delta = \Gamma/\eta$, which is commonly used in film flow analysis, is related to Re_{eq} as follows:

$$\mathrm{Re}_{eq}=\mathrm{Re}_\delta\,\frac{\delta}{\delta_{cont}}. \tag{11}$$

An evaluation of the relation Nu = f(Pr) at a constant Re_{eq} has shown that n = 0.4 is sufficiently accurate.

The relation $Nu/Pr^{0.4} = f(Re_{eq})$ has been plotted in Fig. 5 to a logarithmic scale. The graph indicates that this relation is closely enough approximated (deviations ±20%) by a general power function with exponent m = 0.8 and coefficient A = 0.012. Equation (9) becomes then

$$Nu = 0.012\,Re_{eq}^{0.8}\,Pr^{0.4}. \tag{12}$$

According to Fig. 4, the values of α from (12) agree closely with the experimental data.

NOTATION

x	is the path length traversed by the film, m (mm);
L	is the path length down to where boundary layer becomes film thickness, m (mm);
δ	is the mean film thickness, m (mm);
δ_0	is the film thickness at $x = L_{bound}$, m (mm);
δ_{lim}	is the limit (equilibrium) mean film thickness at $x \to \infty$, m (mm);
δ_{cont}	is the mean thickness of continuous layer, m (mm);
δ_{wave}	is the mean height of wave crest, m (mm);
$(\delta_{cont})_{lim}$	is the stable mean thickness of continuous layer, m (mm);
$L_{0.8}$	is the path length traversed by film, corresponding to $\delta - \delta_{cont} = 0.8(\delta - \delta_{cont})_{lim}$, m (mm);
ν	is the kinematic viscosity of trickling liquid, m^2/sec;
η	is the dynamic viscosity of trickling liquid, $N/m \cdot sec$;
$V = \Gamma/\delta\gamma$	is the mean film velocity at a given section, m/sec;
Γ	is the weight intensity of the trickle, $N/m \cdot sec$;
$Re_\delta = V\delta/\nu = \Gamma/\eta$	is the Reynolds number for liquid film flow;
Nu	is the Nusselt number;
Pr	is the Prandtl number;
t_f	is the local film temperature of trickling liquid, °C;
t_w	is the local wall temperature, °C;
t_h	is the temperature of heat carrier, °C;
$\Delta t = t_w - t_f$	is the local temperature drop from wall to liquid film, °C;
V_{cont}	is the mean velocity in the continuous layer, m/sec;
λ	is the thermal conductivity of film, $W/m \cdot deg$;
α	is the local heat-transfer coefficient, $W/m^2 \cdot deg$;
q	is the local thermal flux density, W/m^2.

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